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Count Riccati's Equation

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"It is Galileo's dictum that mathematics is the language of science. It is more than an aphorism. It asserts that although mathematics and physics are inextricably linked they nevertheless have separate identities and are hierarchically not comparable."[1] Most often the big step for the scientist is the reduction of a particular phenomena under study to a mathematical conceptualization. It is frequently the case that when viewed with hindsight one wonders why, with a little foresight, such a step was not expressly realized long before it actually appeared. One such step that gave, and is still giving, aid and comfort to the physicist and his satellite technologists came in the form of the differential equation, or more generally in the form of a system of differential equations. Unfortunately, in our day to day living, we cannot be satisfied with "just big steps". We must have specifics, and so it is that a solution to such and such a particular differential equation is sought. As is often the case, the equation under consideration is reluctant to yield its hidden wealth by way of a well-manageable "closed form" solution. As it always seems a disproportionate amount of energy is expended in achieving the desired results.

A frequent visitor to the scientist braving the new frontiers (with vigor [but of course, what else]) is the linear second-order differential equation. It came into being soon after the Renaissance with the advent of mechanics and thence later with the rise of electrodynamics. Even today's space age demands of it full measure. However, it often goes almost unrecognized when clothed with today's complicated expressions, and it is these expressions that add to the complexity of formulating a solution. Now it happens that an elementary transformation

reduces this equation to the subject matter at hand, the Riccati equation.

$$\frac{dw}{dz} = f_1(z)w^2 + f_2(z)w + f_3(z)$$

In general this equation as it stands behaves no better than its equivalent counterpart. Its reluctance to yield a well behaved solution has led Professors E. P. Merkes of Marquette University and W. T. Scott of Arizona State University under AFOSR Contract No. AF 49(638)-888 to look deeper into the problem.

Fortunately, the Riccati equation has the formal property that a linear fractional transformation of the dependent variable w with coefficients which are functions of z only yields a new Riccati differential equation in the new dependent variable. An obvious connection with continued fractions arises from the fact that a continued fraction may be interpreted as a sequence of linear fractional transformations. A recent article on the subject has been published by Professors Merkes and Scott. Their article entitled "Continued Fraction Solutions of the Riccati Equations",[2] concerns the general Riccati differential equation with singular point at the origin. Under rather mild hypothesis they get rather striking results as to the nature and form of the solution of such equations.

Professors Merkes and Scott are currently studying continued fractions and the relations these fractions have to partially bounded linear operators, bounded self-adjoint operators and typically real functions.

1.) Bochner, Solomon, The Role of Mathematics in the Rise of Mechanics. American Scientist, Vol. 50, No. 2, June 1962.

2.) Merkes, E. P., and Scott, W. T., Continued Fraction Solutions of the Riccati Equation. Journal of Mathematical Analysis and Applications, Vol. 4, No. 2, April 1962.

PROPAGATION through the Earth's CRUST

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The prospect of propagating radio signals through rock strata within the earth's crust has great appeal to those concerned with military communications. The advantages of such a propagation path are fairly clear. These are low vulnerability to bomb damage, high security, and low ambient noise level.

Several years ago hopes were high that long distance (100 miles or more) underground paths were feasible. Such hopes have now dimmed. For short distance pathways, however, the techniques still hold promise. The advantages of extremely hardened communications even over short paths are of military importance in several applications. This fact has lead AFCRL to make a detailed and rather basic study into the potential of the communication technique.

The AFCRL study has concentrated on the geological, geophysical, and electrical properties of the rock structures. The search for rock strata useful for radio propagation has centered on the Pre-Cambrian rocks of the basement complex. These are the very dense, highly resistive rocks found at surface depth in parts of New England but which reach a depth of more than 25,000 feet in parts of the Southwest.

The basement complex contains both igneous rocks (granites, basalts, gabbros) and metamorphosed sediments (schists, gneisses, marbles). Of primary interest, from the communications standpoint, is the resistivity of these rocks. Low attenuation of the propagated signal requires high resistivity--and this means that the rocks must be free of moisture.

Although the resistivity of the rocks is quite easy to measure in the laboratory, the actual values encountered in <u>situ</u> are considerably different. This is due to the fact that moisture in varying amounts is almost always found in the pore spaces of the rock.

Even dense granite ordinarily considered by geologists to be "dry" often has considerable moisture content.

The AFCRL study has found the geology of the basement rock strata to be more complex than had been suspected. Inhomogeneity of rock type plus faults, fractures, and folding of the strata contribute to a lowering of the basement resistivity from values obtained using a homogeneous model.

Based on electrode methods of electromagnetic sounding used by groups at MIT and the U.S. Geological Survey, it has been found that in most areas of the U.S., basement resistivities will probably be on the order of 10^3 to 10^4 ohm-meters. Resistivities on the order of 10^6 ohm-meters represent the threshold of feasibility in considering the use of subterranean communications beyond a few miles.

The object of the AFCRL study is to define the boundary conditions for the communications scheme. The study to date indicates that for certain locations and for propagation paths of about 10 miles or so the propagation technique may be feasible.

NEW REFRACTOMETER

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A radar looking at a target through the atmosphere is not looking directly at where the target is, but at where the target appears to be. This error is due to atmospheric refraction. Depending upon where the index of refraction gradients are located and how strong they are, the error can become very large. Under some conditions the error may be so large as to lose sight of the target altogether.

The factors that affect refractive index at microwave frequencies are pressure, temperature, and moisture. It is possible to take readings of each of these parameters and from them compute the microwave index of refraction at any point in the atmosphere. But existing procedures are time consuming, require several separate instruments, and lack accuracy. For tracking of missiles and

rockets during launch, a continuous profile is desired and there is no simple and quick way to obtain such a profile. Available instruments for refractive index measurements are much too heavy, requiring an aircraft to carry them aloft, and are expensive, or inaccurate.

There is a need, therefore, for a single instrument to measure pressure, temperature, and moisture simultaneously and indicate the index of refraction directly. This device would have to be light enough and inexpensive enough to be flown aboard conventional weather sounding balloons, and yet be accurate enough to fulfill the strict requirements by the equipment with which it is to be used.

AFCRL is presently developing a refractometer designed to meet all of these conditions. This instrument consists of a dual microwave cavity, one end of which is open to the air. The second cavity is entirely sealed but is connected to the exposed cavity.

In appearance, the instrument has a cylindrical shape--or more precisely it looks like two truncated cones joined together at their base. Length of the proposed model which will operate at about 1000 Mcps is about five inches, and has a center diameter of about one and one-half inches.

The positioning of the cavities along with the fact that the cavities are made from aluminum, insures good heat transfer throughout the instrument. Thermally speaking, whatever happens to one end of the cavity also happens to the other, and by this mechanism both sealed and sample cavities "drift together" in parallel.

The resonant frequency of a cavity depends upon the dielectric constant of the medium within it. Since the microwave index of refraction of the atmosphere is proportional to the square root of its dielectric constant, neglecting any magnetic permeability, the resonant frequency of the cavity is proportional to the index of refraction of the air within it. In this refractometer the air is allowed to enter one cavity, but not the other. Thus, as the frequency of resonance of one cavity changes with respect to the other cavity we have an indication of the magnitude and direction of any refractive index change. It should be possible to calibrate this instrument to read not only changes, but absolute values of index of refraction.

An engineering model will be ready for first tests in June of this year.

AFCRL reorganization

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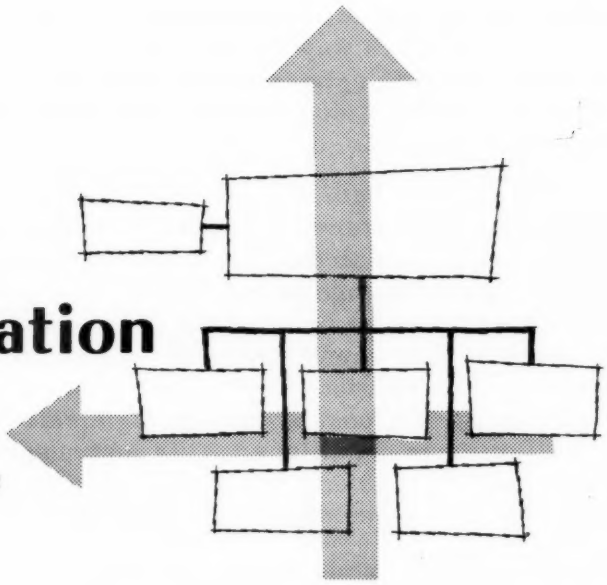

A major reorganization of the Air Force Cambridge Research Laboratories became effective April 15, 1963. The reorganization was taken to reflect the changing mission of AFCRL.

The new organizational structure realigns the varied AFCRL research programs into more cohesive groupings. It does not eliminate or alter any existing programs. The primary objective of the reorganization is to strengthen the overall AFCRL research effort by bringing together related activities previously carried out under separate laboratories. The regrouping will also simplify and improve the management of the research program.

Under the reorganization, the 14 elements formerly comprising AFCRL have been realigned into nine major laboratories. The Electronics and Geophysics Research Directorates, under which the 14 elements were organized, have been formally abolished. The revised laboratory structure represents the first major organizational change of the AFCRL research elements in several years. The changing patterns of research at AFCRL, together with the natural tendency over a period of years for research conducted by one laboratory to expand and overlap similar research in other AFCRL laboratories, dictated the revision of the laboratory structure.

As of 15 April, the following nine laboratories comprise AFCRL:

Terrestrial Sciences Laboratory: This laboratory, headed by Owen W. Williams, is responsible for AFCRL research in the fields of geodesy, gravity, and the earth sciences. The laboratory, its people and its mission, were transferred without change into the new organizational structure.

Meteorology Laboratory: This new laboratory, comprising the former Meteorological Research Laboratory and one element of the former Meteorological Development Laboratory, is responsible for research in the fields of cloud physics, large scale weather patterns, index of refraction measurements, air circulations, and the study of atmospheric constituents affecting weather. Dr. Morton L. Barad heads this laboratory.

Aerospace Instrumentation Laboratory: This laboratory, which evolved most directly from the former Research Instrumentation Laboratory, is responsible for AFCRL's rocket and satellite instrumentation program and for its balloon development efforts. Augmenting the former Research Instrumentation Laboratory will be the AFCRL program in the development of meteorological equipment for Air Force operational purposes. This laboratory is under the direction of Robert M. Slavin and is the largest of the AFCRL laboratories.

Upper Atmosphere Physics Laboratory: This laboratory is responsible for all research into the composition and dynamic processes of the upper atmosphere. A major segment of the effort is concerned with ionospheric research. The laboratory encompasses work formerly carried out under the Detection Physics Laboratory in the field of missile detection involving a disturbed ionosphere. Also under this laboratory is the work conducted by the former Propagation Sciences Laboratory concerned with very low frequency transmissions and with the effects of the ionosphere on high frequency communications. This laboratory is headed by Dr. Christos G. Stergis.

Space Physics Laboratory: This laboratory is essentially a new laboratory, although a major program in radio astronomy was transferred from the former Space Physics Laboratory. Under the new laboratory all of AFCRL's astronomy work is conducted. The

Sacramento Peak Solar Observatory, in New Mexico, and the Sagamore Hill Radio Observatory, in Hamilton, Massachusetts, are a part of this laboratory. The laboratory is also responsible for AFCRL's work in lunar and planetary environments. This laboratory is headed on an acting basis by Dr. Jules Aarons.

Solid State Sciences Laboratory: This laboratory is made up exclusively of elements from the former Electronic Material Sciences Laboratory. It is responsible for the growth and analysis of single crystals of semiconductor and magnetic materials, and for the fabrication of experimental electronic devices. A large segment of the program is concerned with the ultrapurification of materials and for studies into the effects of radiation damage to materials and components. Robert M. Barrett heads this laboratory.

Microwave Physics Laboratory: This laboratory is responsible for AFCRL's antenna research program, for techniques for generating EM energy in the microwave region, and for related plasma physics research. A major segment of this last program is concerned with re-entry communications. The former Electromagnetic Radiation Laboratory, from which this laboratory was formed, has been augmented by programs concerned with EM transmissions through the earth's granitic basement and by research on plasma devices. Carlyle J. Sletten heads this laboratory.

Optical Physics Laboratory: AFCRL's infrared research program is centered in this laboratory. The program is concerned both with the development of improved infrared sensors and with studies of absorption and emission mechanisms of infrared radiation by the atmosphere. This laboratory, except for the addition of programs in ultrahigh temperature research, thermal imaging and laser research is little changed under the reorganization, retaining both the laboratory title and the laboratory Chief, Dr. John N. Howard.

Data Sciences Laboratory: This laboratory was formed out of two laboratories under the previous organization structure, the Computer and Mathematical Sciences Laboratory and the Communication Sciences Laboratory. This new laboratory is responsible for research in speech, in data coding techniques, and for all of the programs formerly carried out under the Computer and Mathematical Sciences Laboratory--microcircuit development, computer languages, biophysics, and logical circuit techniques. The laboratory is headed by Robert M. Alexander.

formula for expression of HARDNESS

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Hardness of solids can be defined quite simply as resistance to structural breakdown or deformity. But this is about the only simple aspect of the subject. Few questions have defied scientific explanation more tenaciously than the question of just what makes some solids hard and other soft. The search for mathematical expression of the factors contributing to hardness in solids is a very old one. Theoretical and experimental work carried out at AFCRL by Dr. J. N. Plendl and Dr. P. J. Gielisse have given us the clearest insight to date on the factors contributing to hardness. These AFCRL scientists have concentrated largely on the study of hardness of nonmetallic crystals.

A reference citation of all those who have considered the question of hardness in solids would extend to impressive lengths. Curiously, the work by the giants in the field has not done much to clarify the situation. There are several reasons for this. Most important of these has been the lack of a common definition of just what constitutes hardness. Another is the absence of a measurable quantity to express the property of hardness on an absolute and comparitive basis. The concept of hardness has thus remained a relative, and in most cases, even a dimensionless quantity. It is subject to various definitions, interpretations, and methods of determination. Most bothersome is the fact that the values obtained by various methods of hardness testing are not directly comparable with one another.

From the formula evolved by Plendl and Gielisse an absolute expression of hardness can be derived. Some 88 nonmetallic compounds, ranging from rocksalt to diamonds, have been examined experimentally with remarkable agreement with theory. The examination of the compounds not only fortified the validity of their hardness expression, but also shed much light on the forces which hold matter together in the solid state.

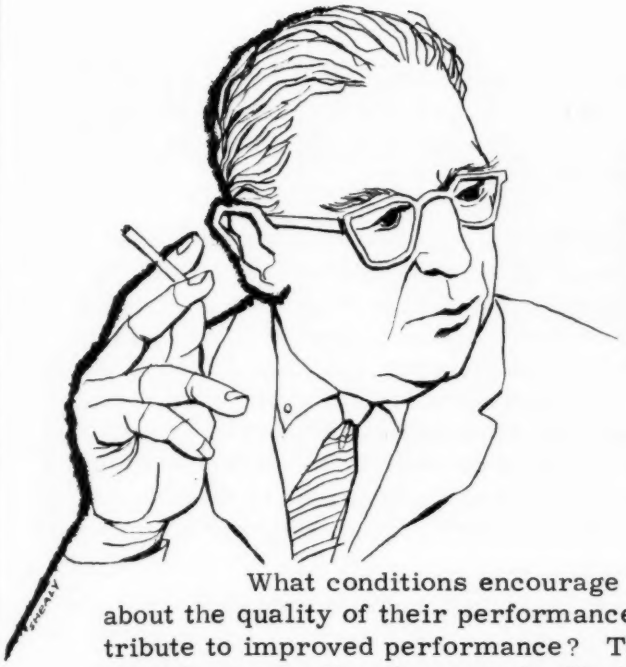
The starting point of their expression, from which the various derivities follow, is the lattice or cohesive energy of the solid and its molecular volume. Cohesive energy may be defined as the energy released when the atomic bonds in a crystal are severed at temperatures of absolute zero. There are several ways for obtaining experimental values of cohesive energy. Values for molecular volume are derived from standard physical tables.

Although cohesive energy and molecular volume considerations are basic to the experession of hardness, the formula itself embodies many considerations. Most important is a subdivision of compounds into one of two classifications which have been termed "hard core" and "soft core". These terms originate from a study of lattice anharmonicity by Plendl.

Still another factor is the non-linearity of the cohesive forces derived from an interrelation between cohesive energy and lattice vibrational spectra--that is, the characteristic vibrations of atoms within the structure which gives each element its distinct infrared signature. From the infrared spectrum, a center frequency (a concept developed by Dr. Plendl) is derived. This is a frequency representing the vibrational spectrum of a solid in its entirety.

The relationship of these and other factors in the expression evolved by Plendl and Gielisse are discussed in their recent AFCRL Research Report (AFCRL-63-26), "Atomistic Expression of Hardness". The formula for expressing hardness in solids is discussed in the report and gives for the first time an unambiguous and strictly comparable hardness scale. The formula also offers certain by-products. If the hardness of a compound is known, for example, it is possible to estimate cohesive energies and the force constants. Hardness "anomalies" such as anisotropy or the increase in hardness recently observed in solid solution crystals of metals and inorganic materials, surface hardening, annealing, and other special treatments, may also be explained in the light of the expression.

Work in this field is continuing. Emphasis is now being placed in obtaining more reliable experimental data over wide temperature ranges.



the supervisor and organizational performance

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What conditions encourage work groups to be concerned about the quality of their performance? How can the supervisor contribute to improved performance? These are questions to which recent organizational research efforts of Doctor Stanley Seashore and Doctor Martin Patchen, social scientists at the University of Michigan under AFOSR support, have addressed themselves.

Studies of industrial production, with regard to quota setting and output, have amply demonstrated that face-to-face groups can exert an enormous influence on the behavior of their members. The question of why a given group adopts a certain standard of performance rather than another standard--say, of holding down output rather than encouraging it--has received much less attention.

Earlier work by the University of Michigan investigators has examined the influence on production of organizational settings which produce confidence in the individual employee that rewards will accompany higher productivity and penalties will accompany lower productivity.

Under AFOSR support, the influence of the supervisor on productivity was investigated with some significant findings. The data indicate, first, that several aspects of a supervisor's behavior must be considered in combination, if one is to understand his impact on group performance norms. In particular, when the supervisor encourages efficiency at the same time he is trying to obtain rewards ("going to bat") for his subordinates, group members appear to react with high standards of performance. It is interesting that actions by a supervisor, which bring about high performance norms under

some circumstances, appear to have the opposite effect under other conditions. For example, encouragement of efficiency by the foreman is associated with lower group performance norms, where the foreman does not "go to bat" for his people.

Perhaps the outstanding findings of the study concern the effects of close supervision. The data indicate that, while close supervision is not beneficial in itself, it can help to produce high group performance in situations where the supervisor is a rewarding figure and where group cohesiveness is high. In the light of previous assertions that close supervision is undesirable, it seems worthwhile to examine more intensively the meaning and effects of "close supervision". The present results suggest that it is beneficial when the supervisor is perceived as a rewarding figure, and a "team member", rather than a punishing outsider.

When the supervisor is interested in the welfare of his subordinates, and is accepted by them as a member of their "team", then his close attention to what his subordinates are doing may be welcomed by them. When the supervisor is around and showing interest in what they are doing, subordinates may be eager to please him and to win his approval. On the other hand, when the supervisor is seen by subordinates as indifferent to their welfare, or even as a hostile outsider, then his close supervision will probably be met with apathy or resentment.

The investigators point out that "close supervision", as defined operationally in their study, means frequent checking on subordinates' work, and not reducing subordinates' freedom to do the work in their own way. They found, as expected, that groups are more likely to develop high performance norms in situations where employees have considerable control over how they do their jobs. It appears then, that checking by the supervisor should take the form of advice and encouragement rather than detailed orders.

It is significant that these findings, based upon data from a large industrial organization, are consistent with those of another Michigan social scientist, Doctor Donald Pelz, who had investigated the performance of scientists in a research organization. He found that scientists produce most effectively when they have freedom to do the work in their own way, and have frequent contact with their supervisors.

MUONIUM ($\mu^+ e^-$)

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To account for nuclear forces, the Japanese physicist Yukawa proposed in 1935 that an additional elementary particle was needed. It was named a "meson", a term which is derived from the Greek and means "medium-weight particle," i.e., heavier than an electron and lighter than a proton. A particle with mass of the order of 207 electron masses was discovered in cosmic rays shortly thereafter. Although this was hailed as the Yukawa meson, later data showed that the new particle, now called the mu meson or muon, did not possess the necessary charateristics to account for nuclear forces. Such a second new particle, slightly more massive than the muon and now called the pi meson or pion, was discovered in the late 1940's and justified Yukawa's prediction.

Although the muon was a flop in its early life, because of mistaken identity, it is now known as a particle with interesting characteristics of its own. The muon possesses many properties which are similar to those of the electron. It exists in both a(+) and (-) charge state, it has an intrinsic spin quantum number of 1/2, it is a fermion and it takes part in weak interactions. In fact, it is so similar to the electron except in mass that many physicists wonder why it exists at all.

The results of the parity-violation theory and experiments over the past few years have made possible the discovery and study of a new hydrogen-like atom composed of a positive muon (μ^+) and an electron (e^-). This new atom is called muonium and possesses an energy level scheme similar to hydrogen when account is taken of the proton-muon mass difference. As is the case in hydrogen, muonium possesses a hyperfine structure to its energy level scheme due to the interaction between the magnetic moments of the muon and electron. In hydrogen, the hyperfine structure separation of the ground state is 1420 Mc/sec, while in muonium it was predicted to be of the order of 4500 Mc/sec.

Professor Vernon Hughes, under an AFOSR grant at Yale University, is presently engaged in an experiment designed to make precise measurements of the hyperfine structure of muonium in order to determine whether the electron and muon differ in any respect other than mass. Since the mean life of the muon is only 2.2 microseconds, the work is not an ordinary experiment in spectroscopy. Using the Columbia University Nevis synchrocyclotron, which accelerates pro-

tons to 380 Mev, protons are used to strike nuclei and produce pions. The pions (π) then decay to muons (μ) and neutrinos (ν).

$$\pi^+ \longrightarrow \mu^+ + \nu$$

As a consequence of parity non-conservation in this decay, the muon is produced with its spin vector antiparallel to its linear momentum. In other words, it is polarized antiparallel to its momentum. The muonium can now be formed when the μ^+ captures an electron from the background gas - in Professor Hughes' experiment it is argon (A).

$$\mu^+ + A \longrightarrow (\mu^+ e^-) + A^+$$

Since the muon was initially polarized, the muonium is also polarized. The muon then decays with its characteristic mean life.

$$\mu^+ \longrightarrow e^+ + \nu + \bar{\nu}$$

The e^+ is a positron or anti-electron and the symbol $\bar{\nu}$ stands for the anti-neutrino or anti-particle to the neutrino. By now observing the angular asymmetry of the positrons from the decay of the muon, the state of the muon spin in muonium can be determined. The positrons can have an energy up to 50 Mev and can be easily detected even though their number is small.

The hyperfine structure is studied by inducing transitions in the muonium before it decays. By introducing microwave energy into a cavity with the correct frequency for a transition ($\frac{\Delta E}{h}$, where ΔE is the energy difference between states and h is Planck's constant), such transitions can be induced. The effect is to change the energy state of muonium by flipping the spin of the muon. The change is observable from its effect on the angular distribution of the positrons.

With the described technique, Professor Hughes and his collaborators have been able to measure the hyperfine structure of muonium to one part in 10,000. Their data agree with the theoretical predictions thus furnishing experimental evidence that the muon indeed behaves like a heavy electron.

Under a continuing grant providing joint support by AFOSR and ARPA, Professor Hughes will investigate the chemical reactions of muonium. The reason for such interest is the discovery in the earlier experiments that a small amount of air mixed with the argon causes the loss of polarization in muonium. Hughes feels that the muonium is forming a molecule with oxygen or nitrogen and plans to determine the cross sections for such reactions.

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